A Phenomenological Model of Parameter Growth in Engineering Systems

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Abstract

A new model for approximation and prediction of growth of the parameters of engineering systems is suggested. The model derives the rate of growth not only from the considered system itself but also from the customers' expectations that play a role similar to the "driving force" in thermodynamics. The suggested model is written in the form of the system of few differential equations that can be solved by numeric calculations, similarly to the simulation of the structural and stress relaxation phenomena in super-cooled liquids. Some examples of applications of the simplified model are presented.

Keywords: engineering system, technical system, growth, parameter, S-curve, model, relaxation

1. Background

The Theory of Inventive Problem Solving (TRIZ) believes that development of the Engineering Systems (ES) can be considered as an evolutionary process that undertakes some general laws. The term "engineering system", in our consideration, means a "population" of particular systems that satisfy a particular human need in similar way like "car" (for transfer by road), "aircraft" (for transfer by air), "photo camera" (for making a picture by using optical lens), etc.

Although exact formulations of the laws of evolution of engineering systems are not known yet, many rather common trends describing the development of multiple particular systems seem to confirm the existence of such laws. For example, various engineering systems (transport, weapons, information systems, etc.) being developed become more powerful, more "dynamic", better-controlled, less human-involved, etc. (Leon, 2006).

Multiple investigations describe the progress of particular kinds of systems in terms of the "evolution" of the "key parameters", e.g. speed and range for transport, resolution and sensitivity for photo cameras, power and weight for batteries, etc. (Martino, 1972 & Kynin, 2009). Such consideration generally leads to a concept of so-called "S-curve" (Fig. 1). There are several different kinds of concepts describe "S-like" dependences of various quantities from each other. In this paper, we will consider only one of them: the dependences of the key parameters of a system on time. This kind of dependences is often called a "life line".
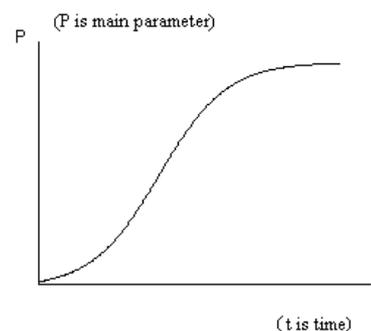
Fig. 1. Schematic view of S-curve

In the above-mentioned and other investigations, it was shown that the concept of "S-curve" can be well applied as a rough approximation of various "life lines". So, it was natural to try to describe these (generally similar) curves by some mathematical expression to be able

to quantitatively predict the growth of key parameters of a system in the future. Indeed, knowing the "life line" of a system could allow developing of an ES with maximum effect for minimal cost that is a basic idea for the "Directed Evolution" concept (Zlotin, 2001).

For that, multiple approaches were proposed. Bass (1969), Kohlrausch (1863), Modis (1992) etc. described multiple more or less simple equations that can more or less correctly describe the growth of various characteristics of ES. In this way, however, serious problems appeared.

First, not all of the systems (and surely not all of their key parameters) demonstrate the S-like behavior. Kynin (2010) mentions that we considered the examples of abrupt halts and rather long delays in the "life lines" of key parameters of multiple systems. These events are usually very difficult to forecast. As a demonstrative example, we can mention serious problems of the Microsoft Corporation with the OS Windows Vista. This operating system has been developed under the assumption that the growth of the clock rate of PC processors described by nearly exact exponential curve (twice growth each ~18 months) during several decades could be extrapolated for the next few years. However, after reaching about 3 GHz, this characteristic suddenly stopped its growth, and now we mostly use the processors with virtually the same clock rate as it was 3-5 years ago. The "physical barrier" has been achieved instantly and manifested as a break point instead of expected "smooth" slowing the growth rate. As a result, the operating system optimized for 10-20 GHz did not satisfy most of customers having "slow" 2-3 GHz computers.

Second, sometimes the market seems not "to like" the improvements of the systems and does not accept the products with seemingly quite better characteristics. This situation can be considered as opposite to previous one. For example, it is easily possible to develop a locomotive with the speed of ~250 km/h able to be used in conventional railroads. These locomotives really exist and even are commercially used somewhere, e.g. on the route Moscow – St. Petersburg in Russia. However, the typical maximum speed of conventional locomotives is now nearly the same as it was half a century ago: somewhat about 120 km/h. Improvement of this characteristic is accepted by market only for the next generation of the railroad transport that uses other types of ways (Kynin, 2011). In this case, investments to a system did bring the expected technical result – but this result was not required by the infrastructure. This situation is known in the scientific world as "good enough".

Sometimes, however, the key parameters of a system which typical values did not change for long enough time (see above), suddenly start to grow again. Kynin (2011) considered several examples of this kind and concluded that such behavior can often be explained as a result of competition with a new system with potentially better characteristics.

The above-described kinds of behavior of the "life lines" seem not to be predictable by using the mentioned approaches that consider only a system itself, without its competitors and surroundings. Below we suggest a new approach that allows, at least theoretically, description of above-mentioned behavior of the "life lines". However, in the simplified model described below we consider only the "regular" case i.e. growing the main parameter of a system to some limit.

The idea of simulation is borrowed from chemical thermodynamics where any process is described in terms of current state, external conditions, target state, driving force and internal parameters ("order parameters"). Certainly, for an engineering system it is, in general, very difficult to apply this approach in its "pure" form because even if we define all required parameters of a considered system and all of its competitors it might be very difficult (if possible at all) to exactly determine their numeric values. However, there is a simulation that can quantitatively describe the behavior of complicated systems with big (or even infinite) number of "order parameters" under arbitrary external conditions. This is the simulation of structural and stress relaxation in in amorphous materials like glasses and polymers (Scherer, 1986).

Within the frames of this approach, the process in a system is described in terms of equilibrium and non-equilibrium states according to Le Chatelier's principle. The equilibrium state of a system is determined as a state having no tendency to change in time under given (constant) external conditions. Each particular combination of external conditions corresponds to one and only one equilibrium state of the system. In each particular equilibrium state, the system has constant characteristics (physical properties) that do not depend on the way of achieving it. Any change of the external conditions causes so-called relaxation process in the system that tends to come to new equilibrium state corresponding to new conditions. Multiple changes of the external conditions cause multiple responses, each of them being independent of all others (superposition of responses). Changes of multiple external conditions cause multiple responses independent of each other (superposition of excitations). Each relaxation process in the system is a

linear combination of multiple "particular" relaxation processes, each of them being characterized by a single internal parameter of the system that determines the time scale of the process ("relaxation time"). Relaxation times of particular relaxation processes depend on a single integral characteristic that can be represented as a function of all internal parameters of the system (cooperative change of relaxation times). Each particular relaxation process is governed by its "driving force" determining the deviation from equilibrium state and relaxation time determining the rate ("speed") of coming to equilibrium.

2. Mathematical description of the model

2.1 Basic equations of the relaxation model

Mathematically, the model of structural relaxation can be represented as a system of the following equations:

$$P = f_p(X, X_f) \cong P_0 + \alpha_f X + (\alpha_e - \alpha_f)\sum_{i=1}^{n} g_i(x_i - X); \quad (1)$$

$$X_f = \sum_{i=1}^{n} x_i g_i, \quad \sum_{i=1}^{n} g_i = 1; \quad (2)$$

$$\frac{\partial x_i}{\partial t} = -\frac{X - x_i}{\tau_i}; \quad (3)$$

$$\tau_i = K_i \tau_0; \quad (4)$$

$$\tau_0 = f_\tau\left(X, \sum_{i=1}^{n} x_i g_i\right). \quad (5)$$

Here P is a characteristic (property) of the system to be measured. X is an external parameter which change (excitation) starts the relaxation process (response), and x_i are internal parameters (i is an index of a particular parameter corresponding to a particular relaxation process. n is the number of particular processes); g_i are "weight factors" of particular processes determining their contributions to the "macroscopic" state of system. t is time; τ_i are relaxation times of particular relaxation processes; τ_0 is mean ("weighted") relaxation time that can be considered as a macroscopic characteristic of the system. f_p and f_τ are some functions (in particular, they might be assumed the same that simplifies calculations). K_i are ratios of particular relaxation times to τ_0 (it is assumed that the K_i values are constants). α_e and α_f are constants determining (as a first approximation) the rate of change of the P value after very slow and very fast changing of X parameter correspondingly. In the first case, the system can be considered as approximately equilibrium, and in the second case as nearly "frozen" (that describes the indexes "e" and "f" near X).

In equilibrium state, $x_i = X$, $\partial x_i / \partial t = 0$ for all x_i, and, correspondingly, $X_f = X$; $\partial P / \partial t = 0$ for all P that means that the state and all measurable characteristics of the system are kept unchanged for unlimited time while the value of X is kept constant.

If the system is in equilibrium state and then at the moment $t=t_0$ the X value instantly changes from X_1 to X_2 and then is kept constant (i.e. $X = X_1$ at $t<t_0$; $X=X_2$ at $t \geq t_0$) the relaxation process in the system begins at t_0. According to Eq. (3) it is assumed that particular relaxation processes at constant values of X and τ_0 can be described by exponential equations:

$$x_i(t) = X_2 + (X_1 - X_2)\exp\left(-\frac{t - t_0}{\tau_i}\right). \quad (6)$$

Accordingly, the P value (i.e. the characteristic that we are interested in) changes as

$$P(t) = P_0 + \alpha_f X_2 + (\alpha_e - \alpha_f)\sum_{i=1}^{n} g_i(x_i - X_2) =$$

$$= P_0 + \alpha_f X_2 + (\alpha_e - \alpha_f)\sum_{i=1}^{n} g_i(X_1 - X_2)\exp\left(-\frac{t - t_0}{\tau_i}\right). \quad (7)$$

According to principle of superposition of responses, gradual change of X can be approximately presented as a series of "instant" changes and consequent time intervals where the X value remains unchanged. Thus, it is possible to calculate the value of P for any "profile" of changing of X with time by using numerical methods.

2.2 Relaxation model for engineering systems

Now let us try to find the equivalents of the parameters of structural relaxation model in the development of engineering systems. The general scheme of the considered factors is shown in Fig. 2. Here P is the main parameter of the system (to be predicted), and X value corresponds to the "equilibrium state" (the value that could completely satisfy the customers).
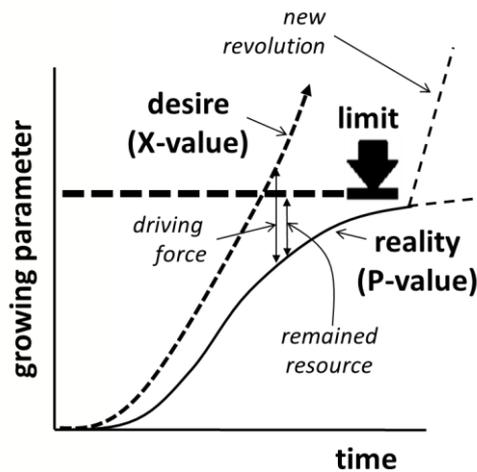
Fig. 2. Schematic view of factors considered by the model

We do not know exactly which characteristic can be considered as the main parameter for a given system. In terms of TRIZ, this characteristic can be presented as an analog of "ideality" that is determined as a ratio of "sum of profits" to "total cost" where the latter one includes cost and harmful side effects caused by the development, production, use and utilization of the system. It is also possible to consider the reciprocal characteristic: acceptable cost of a unit of the main parameter of the system. Thus, the X value can be represented as AC/MP or MP/AC where AC is Acceptable Cost and MP means the value of the Main Parameter of the system. For example, if the main parameter of a digital camera is its resolution (that was actual few years ago) then X value would be determined as acceptable cost of the resolution of 1 pixel of a picture, or as the number of pixels acceptable for 1 dollar of the cost of camera. It is also possible to use the logarithmic scale, namely:

$$P \equiv \log(MP / AC) = \log(MP) - \log(AC) \cdot \qquad (8)$$

To judge if this assumption is correct or not we need additional investigations. However, even without clarification of this or other choice, we can assume that *some external characteristic* plays the role of *X*-value. Thus, we can try to fit this value and its growth empirically without loss of generality of the model.

The sense of the "internal" parameters of the system (x_i) is most difficult to realize in the structural relaxation model. No exact meaning of these parameters exists. The structure relaxation model does not require ascribing exact sense to these parameters: it is quite satisfactory to determine them empirically as "something in the system".

Now let us focus at the relaxation times τ_i. These parameters determine the time required for decreasing the *F* value for *e* times ($e \cong 2.718$ is the base of natural logarithms) under the constant *X* value, i.e., in our case, increasing the main parameter of the system for *e* times. It is known that in some practical cases this value is approximately constant for rather long time. For example, for various characteristics of computers (clock rate of main processor, RAM memory size, typical capacity of the hard drive, etc.) this situation was observed during about half a century, from 1950th to the beginning of 2000th (the so called Moore's law); the typical τ values were about 2-4 years all this time. However, for particular engineering systems this characteristic is usually not constant and tends to increase with time. According to Leon (2006), exponential growth is only an extreme scenario of evolution that is normally observed only in the beginning of the system evolution. As far as the system comes nearer to its physical limit of growth, the "normal" relaxation time increases, with a tendency to infinity when this limit is achieved. So, let us suggest the simplest equation that describes this behavior:

$$\tau_i = \frac{K_i}{\log(X_{i,\max} / X_i)} = \frac{K_i}{\log X_{i,\max} - \log X_i} \qquad (9)$$

where K_s is normally a constant for a given system, and X_{max} is the limit (or barrier) for the *X* value.

It is known however that big enough investments can greatly accelerate the progress of the system, i.e. to reduce the relaxation time τ_0. Let us implement a new term ω to Eq. (9) to consider the effect of investments:

$$\tau_i = \frac{K_i}{\left(\log X_{i,\max} - \log X_i\right)^\omega} \cdot \qquad (10)$$

The quantity ω should demonstrate the following behavior:

$$\begin{cases} \omega \approx 1 \text{ at no special investment;} \\ \omega \to 0 \text{ at maximum possible investment.} \end{cases}$$

This means that at normal investment we have "natural" growth curve (with decreasing the rate with time), and at maximal investment the growth rate is kept nearly constant, only slightly depending on the difference between *X* and X_{max}. The simplest expression having these properties is

$$\omega(t) = \frac{k_\omega}{\exp(dI / dt)}, \qquad (11)$$

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where dI/dt is the invest to the considered system per a unit of time, and k_ω is a constant. However, for short enough periods of time, we can neglect the time dependence of ω considering it as a constant for a given system.

Then we have to specify the expression for K_i. From very general consideration, we know that normally the rate of technical progress exponentially increases with time (Wikipedia, 2014). In terms of our model, it means that the values of K_i can be considered as functions of the total time starting from the first (working) appearance of the system:

$$K_i = \exp\left(-\frac{\Delta t_i}{K_s}\right), \qquad (12)$$

where Δt_i is the time difference between the moments when the system itself and a given kind of this system appeared, and K_s is a constant for the system. For example, in a system with $K_s = 1$ year the relaxation time of each new "generations" will be diminished for e times every year.

3. Simplified model and its practical application

3.1. Simplification of the model

The above-described model is able (at least, in principle) to describe various complex scenarios of the development of Engineering Systems. For that, one needs to determine the parameters that describe the considered system itself, its competitors and the surroundings.

However, we consider it reasonable to start the verification of the model with the simplest (but important for practice) case of behavior: monotonic growth without halts as depicted in Fig. 2. We believe that if a model of some phenomena is correct then the simplest behavior should be properly described by the simplest case of the model. So, let us try to simplify the model in maximum possible extent and then try to apply it to the description and prediction of monotonic growth of the engineering systems.

The main simplification is to drop multiple relaxation processes off and to consider only one of them. This simplification turns the system of equations (1-5) to the following form (as far as we have only one relaxation process, the indexes i are also dropped):

$$
\begin{cases}
P = P_0 + \alpha_f X + (\alpha_e - \alpha_f)x; & (13) \\
\dfrac{\partial x}{\partial t} = -\dfrac{X - x}{\tau}; & (14) \\
\tau = \dfrac{K_s}{(\log x_{max} - \log x)^\omega}. & (15)
\end{cases}
$$

Then let us consider the P value being the main parameter of the system, without its attribution to cost. This simplification is equivalent to the assumption that the cost of the unit of the main parameter changes much slower than this parameter itself.

Next, let us assume that $\alpha_f \ll \alpha_e$, i.e. that the changes of x-value in the "frozen" state are negligible. This assumption allows dropping the X-value from Eq. (13):

$$P = P_0 + \alpha_f x. \qquad (16)$$

Correspondingly, the X-value becomes some external function that can be considered independently of other characteristics describing the system itself. We can assume that for the systems demonstrating similar behavior, the time dependences of X-values would also be similar. As far as we postulate this similarity, it becomes natural to reduce them to some universal "master curve". For that, we consider it reasonable to introduce some "reduced parameters" t_r and P_r instead of time t and growing parameter P:

$$t_r = (t - t_0) / K_s; \qquad (17)$$

$$P_r = (P - P_0) / (P_{max} - P_0); \qquad (18)$$

$$\tau_r = \tau / K_s, \qquad (19)$$

where t_0 is the moment of time when the system was appeared, P_0 is the value of P-parameter at this moment, and P_{max} is the maximum value of P (i.e. the limit of the development).

Last, as far as we consider the P value proportional to x there is no more need to use the last value. We can rewrite all equations directly substituting P_r for x. After all mentioned substitutions we finally have:

$$
\begin{cases}
\dfrac{\partial P_r}{\partial t} = -\dfrac{X - P_r}{\tau_r}; & (20) \\
\tau_r = \dfrac{1}{(-\log P_r)^\omega}. & (21)
\end{cases}
$$

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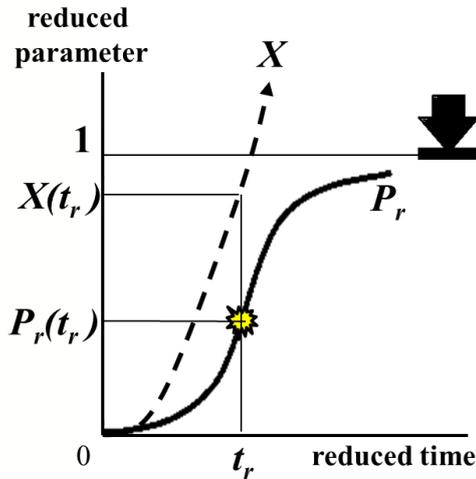After reduction, the model can be presented as shown in Fig. 3.

Fig. 3. Schematic view of the reduced model

The difference $(X – P_r)$ corresponds to the "driving force" of the relaxation process, and the difference $(1 – P_r)$ describes the remained resource of the main parameter's growth.

The S-shaped form of the curve calculated by the model is caused by the specific change of the derivative $\partial P_r/\partial t$ according to Eqs (20) and (21): both the numerator and denominator of Eq. (20) continuously grow with time but the growth curves are different as shown in Fig. 4.
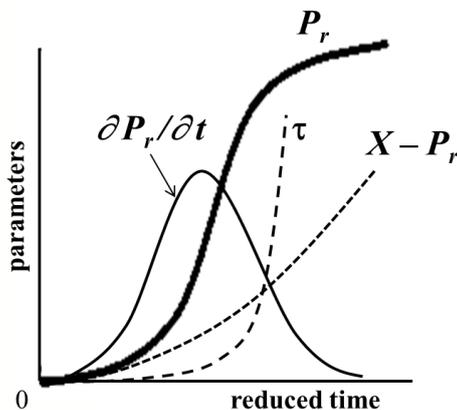

Fig. 4. Schematic explanation of S-shaped growth

Thus, we only need to determine the function $X = f(t_r)$ to be able to perform practical calculations.

After some trials, we came to conclusion that for most practical S-curves, one of the following two equations for X-function is applicable:

$$X (t_r) = t_r \tag{22}$$

or

$$X (t_r) = \exp(2t_r – 3.67). \tag{23}$$

The functions (22) and (23) describe the S-curves with fast and slow starting parts correspondingly.

The equations (17-23) allow calculation of the growth curve for an arbitrary growing variable P for which the following values are known:

- P_0 and P_{max}: minimum and maximum values (in practice, the P_0 value can often be considered as zero);
- t_0: moment of time when the considered system appeared;
- K_s and ω: parameters of the model;
- Kind of X-function: either (22) or (23).

Detailed description of the calculation algorithm is presented in the Appendix 1.

3.3. Data processing

When applying the model, three questions arise: (1) how to find the main parameter of the system, which is usually a complex parameter, i.e. some relationship of particular parameters of the system (Kynin, 2009). (2) how to determine the limit of growth P_{max} in advance, and (3) how to determine the model parameters K_s and ω.

Our approach determines the main (complex) parameter of the system (Priven, 2011 & 2012). Shortly, as far as we consider the main parameter as *something that customers are ready to pay for*, it is natural to expect significant positive correlation of this parameter with the total cost of the system. In a particular case when the major constituent of the total cost is the cost of the product, we can expect significant positive correlation between the values of the main parameter and the market prices of best-selling items. An example of such correlation for laser printers is presented in Fig.5. (We consider only mid-price range: for the cheapest printers the market price cannot be considered as a main constituent of the total cost whereas the most expensive printers are not in competition with considered ones.) Some other requirements to complex parameter are considered in Priven's work (2011).

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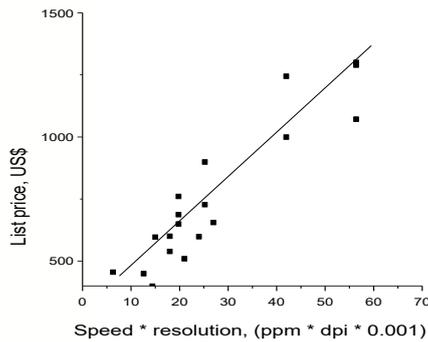
Fig. 5. Correlation between the main parameter of laser printers and market price (for the range from 400 to 1500$) of the best items according to five independent consumers' and experts' ratings (see refs in Priven, 2011 & 2012).

Fig. 6. Results of simulation by the suggested simplified model with different values of parameters:
1: $X = t$; $\omega = 1$; 2: $X = t$; $\omega = 0.7$; 3: $X = t$; $\omega = 0.1$; 4: $X = $ exp(-3.7 + 2 t); $\omega = 1$; 5: $X = t$; $\omega = 0.7$. In all cases, we set $K_s = 1$, $\alpha_f = 0$; $\alpha_e = 1$; $x_{1,\max} = 1$; $P_0 = 0$.
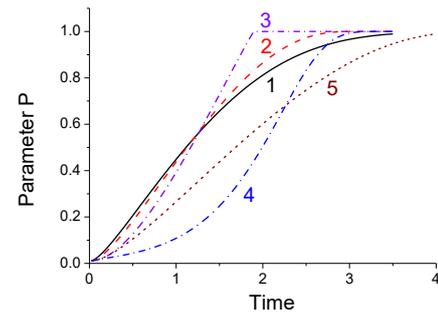
The limit of growth P_{max} can be determined in two ways: as one of the fitting parameters of the model or from some external reasoning, such as physical limits for a given operation principle. Detailed consideration of the problem requires a special publication. Shortly, as far as the model contains only two fitting parameters it is possible to add P_{max} as the third fitting parameter and find its value together with K_s and ω, e.g. by least square method.

After the complex parameter of the system is found and the parameters of the model are determined, it is possible to perform the calculations as described above.

3.4. Examples of practical application

In Fig. 6, the results of numerical simulation with five combinations of parameters ω and $X = f(t)$ are presented. Below we consider some practical situations where the "life lines" have shapes of these curves.

From this picture, we can see that the model, even after drastic simplification, is able to describe various forms of the growth curves including such effects as diverse asymmetry, fast and low start, abrupt stopping the growth, etc.

Below we demonstrate some examples of comparison of the model with factual data (Sood, 2005&2009, Tsao, 2004, Intel, 2011, Brodrick, 2013), the data shows the development several systems. The source data are presented in the Appendix 2.

In Fig. 7, growth curves of the key parameters of CRT monitors (product of diagonal size and resolution) and dot matrix printers (product of printing speed and resolution) are presented. These variables were selected as complex parameters describing the most important characteristics of corresponding systems, which are the Main Parameters of Value in terms of TRIZ (Efimov, 2011).

The considered systems were selected as far as their evolution is now virtually completed (they exist only in the narrow market niches), so that it is possible to overview the whole curves. However, for determination of the model parameters, we used only the starting (left) parts of both curves; the right parts were used for model validation. The value of P_{max} was considered in this case as a fitting parameter; in Fig. 7b, we demonstrate several variants of such fitting for several different starting segments of the curve. As we can see from the figure, the results of calculation (presented the Table 1 of the Appendix 2) are in good accordance with actual data.
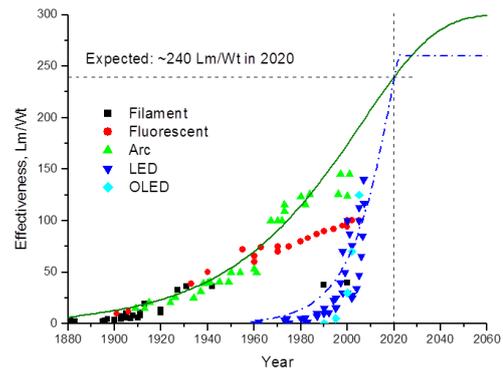
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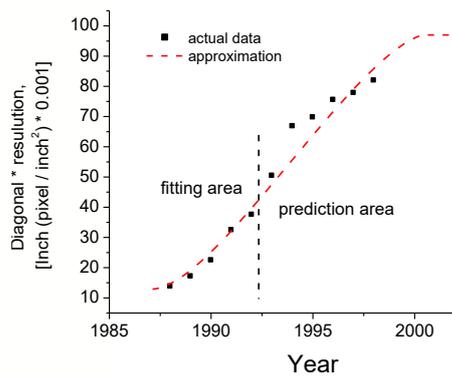
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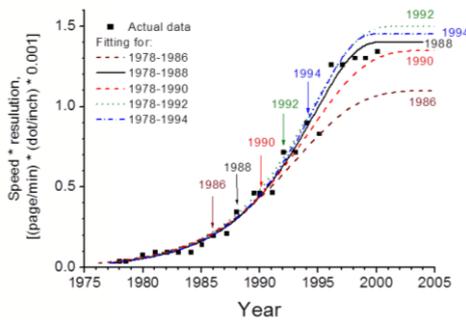
b

Fig. 7. Growth of the key parameters of CRT monitors (a) and dot matrix printers (b) (Sood, 2009).

Fig. 8. Growth of the effectiveness of the electric lamps (Tsao, 2004)

Fig. 9. Growth of the clock rate of personal computers (in semi-logarithmic scale) (Intel, 2011)

In Fig. 8, we made some prognosis for the growth of the effectiveness of the electric lamps.
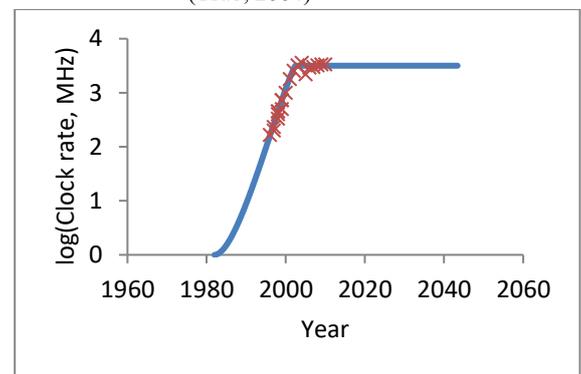
Like previous picture, markers mean actual data and curves show the results of simulation. Judging from our modeling, we expect maximum effectiveness of lamps come to approximately 240 Lm/Wt in 2020.

3.4. Specific case: simulation of the "Moore's law"

In Fig. 9 we consider growing the clock rate of processors of personal computers (PC) starting from the appearance of Pentium processor. In this case, we used logarithmic scale that is only available for prediction in the case when a well-developed system continues huge growth. In this case, the model predicts abrupt stopping the growth. The exact value at which the growth is to be stopped is not predictable by the model; however, the model properly predicts that no "precursors" of such behavior occur (see above). The actual data are in good accordance with this prediction.

Let us describe this example in more detail. As it was stated above, this engineering system demonstrated exponential growth of their characteristics that is often called "the Moore's law" (Hutcheson, 2005):

$$\frac{dP}{dt} = k\,P; P \sim \exp(t - t_0) \cdot \qquad (24)$$

Considering the fact that the average cost of personal computers very slightly changes with time (a contemporary notebook with middle characteristics costs nearly the same as PC XT twenty years ago), we can conclude that the "life line" shown in Fig. 5 can be applied to the acceptable cost as well. Let us remind that the P value in this case means the *logarithm* of the clock rate.

Such behavior can be simulated by the suggested model (again, with a single relaxation process) in the case when the relaxation time τ_0 and the term $(X - P_r)$ in Eq. (20) remains constant, regardless of the distance from the current P_r value to the limit ($P_{r,max} \equiv 1$). The latter feature corresponds to $\omega \cong 0$ in Eq. (21). This specific situation means that whatever the value of the main parameter of the system would be, the surroundings

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(customers, infrastructure, etc.) requires its further growth, regardless of the cost of its improvement. In fact, this situation is close to the struggle against a global threat when to survive is much more important than to save cost. Indeed, we can observe such situation in the history of wars.

However, the above-described situation can also be made artificially, without any external threat. For that, we only should assume that the system, for some reason, has unlimited resources. Now let us ask ourselves a question that is commonly used in so-called "failure analysis approach" (Kaplan, 1999) widely known in TRIZ: how to force the environment (i.e. human society including manufacturers, buyers, governments etc.) not to spare the resources for a system? As far as we formulate this question, the answer becomes obvious: somebody in the surroundings should get benefits that grow with growing the key parameter of a system. In essence, it means the positive feedback between the activity of the system and the environmental benefits; in other words, there must be a couple of synergists that amplify each other as a result of their activity.

If we consider development of personal computers from this viewpoint, we can easily find that the growth of the key parameters of computers really gets additional benefits to the computer industry, and the software developers play the role of synergists for hardware producers. Indeed, the more resources the hardware gives the more complicated – and more convenient – new software can be developed for this hardware. This stimulates the further growth of the characteristics of the hardware in turn. Realizing this situation, the software developers did the next step: they artificially stimulated hardware producers to increase the resources more and more to accelerate the "aging" of the software. This meant that the computer programs that essentially satisfy most of customers *do not work* in new computers because of the lack of resources. Thus, the customers who want to use the contemporary (the most convenient) software are forced to update the hardware, after which they need new version of the software in turn. Actually, the cost of the R&D in the computer industry ceased to play a role of the "bottleneck", i.e. the hardware developers had virtually "infinite" resources for improving the hardware.

If we now return to Eq. (21) above then we come to a conclusion that as far as the main parameter of a system approaches to its limit the base of the denominator of the right part of this equation tends to zero: $\left(\log P_{r,\max} - \log P_r \right) \rightarrow (\log 1 - \log 1) = 0$, and even small power index ω cannot prevent fast growth of the

relaxation time τ. In other words, the growing process should be rapidly stopped near to the mentioned limit.

The Fig. 5 shows that this is exactly what we observe: the growth of the clock rate rapidly stopped near 3 GHz after exponential growing during more than 20 years.

4. Relaxation model and Synergetics

Now it is common knowledge that the evolution of self-developing systems can be described within the frames of synergetics. This branch of knowledge has been first developed by Ilia Prigogine and its coworkers. (Prigogine, 1969) The "synergetic" ideas are now widely used for descriptions of evolution in various kinds of natural and artificial systems. We believe that the basic concepts of the synergetic theory can be applied to the evolution of the engineering systems as well.

In this connection, we have to note that the structural relaxation model that we used as a base for the suggested model uses the concepts that are essentially similar to synergetics. Volkenstein (1956) & Mazurin (1986) mention the structural relaxation was considered within the frames of thermodynamics. From these works, one can conclude that the phenomenon of structural relaxation simulated by this model concerns essentially the same kind of behavior as synergetics does: thermodynamically non-equilibrium state.

In our previous paper (Priven, 1987), it was shown that under some external conditions the structural relaxation demonstrates the behavior which is similar to the synergetic systems: in particular, it becomes difficult to exactly predict because of appearance of positive feedback between excitation and response. In the present model, such positive feedback also can appear that we showed in the last example.

Above we showed that multiple features that can be observed in the "life lines" of real engineering systems can be easily simulated by using an essentially very simple model that is based on very common assumptions (such as superposition, equilibrium, relaxation, etc.). We believe that the further development of the model would help to describe and predict multiple features of behavior of engineering systems that are now very difficult to simulate and properly predict.

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5.   Conclusion

A new model of evolution of engineering systems is suggested on the base of a known model that describes the phenomenon of structural relaxation in amorphous physicochemical systems. This model uses the concepts of equilibrium and non-equilibrium states of a system, Le Chatelier's principle of tending of a system to equilibrium, multiple particular (elementary) processes impacting the "visible" results, superposition and cooperation of these processes.

The model is (at least, in principle) able to explain some specific behavior of engineering systems that has been observed in practice and (in our opinion) unlikely can be properly described within any of existing models and approaches taken one by one.

To compare the model predictions with actual data we considerably simplified the model. Although the simplified model cannot predict all possible cases of behavior of the growing system, it is considerably more flexible than the known simple models. At that, the simplified model contains only three fitting parameters. One of them is the maximum value of the growing variable that can be either used from external data or fitted as a model parameter. In the first case, the number of fitting parameters reduces to two that is the same as in the mentioned models. However, containing only two fitting parameters our models is able to properly describe and predict various cases of growth curves, including different asymmetry and different shapes of starting and ending parts. In particular, the model properly predicted the fact of abrupt stopping the growth of the clock rate of the processors of personal computers after long years of exponential growth, without any "precursors" of such behavior.

We believe that the suggested approach could be helpful for forecasting the evolution of engineering systems including specific cases which are difficult to explain and simulate by using the known approaches.

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APPENDIX 1: Algorithm of calculation

<u>Step-by-step calculation</u>

As mentioned above, the calculations are performed step by step. Each step consists of instant growth of X value followed by the relaxation process at which the internal parameter x (that was reduced to the P_r value) gradually comes to the equilibrium.

Our practice showed that the optimal value of the time step is

$\Delta t = 0.01\ K_s$,

which corresponds to the step of the t_r value equal to $\Delta t_r = 0.01$.

The calculation procedure consists of the following parts:

(1) We postulate some initial conditions;
(2) We make the first step by a simplified algorithm;
(3) We make the next steps by using normal algorithm;
(4) We exit calculations when the P value becomes close to saturation.

Below these parts are considered in more details.

<u>Initial conditions</u>

On start ($t = t_0$), we accept the following values of variables:

$t_r = 0$; $P_r = 0.0001$; $P = P_0 + 0.0001\ (P_{max} - P_0)$.

The value $P_r = 0.0001$ instead of 0 is accepted for simplicity of further calculations. This does not affect the final result as far as the error of calculation is anyway much greater than 0.01%.

<u>First step of calculation</u>

Below, the subscript "1" means that the corresponding values concern the end of the first step, i.e. the moment of time corresponding to $t_r = 0.01$; $t = t_0 + 0.01\ K_s$. At this moment, we have:

$t_{r,1} = 0.01$;
$P_{r,1} = 0.0001$;
$t_1 = t_0 + \Delta t = t_0 + 0.01\ K_s$;
$P_1 = P_0 + 0.0001(P_{max} - P_0)$.

<u>Next steps of calculation</u>

Below, the formulas for the next steps are presented. All values concern the end of the corresponding step of calculation; the number of step is specified as "i":

$X_i = X(t_{r,\ i-1})$;
$\tau_i = (-1/\ln P_{r,\ i-1})^{\omega}$;

$(dP_r/dt_r)_i = - (X_i - P_{r,\ i-1})\ /\ \tau_i$;
$P_{r,i} = P_{r,\ i-1} + (dP_r/dt_r)_i\ \Delta t_r$;
$t_i = t_{i-1} + \Delta t = t_{i-1} + 0.01\ K_s$;
$P_i = P_0 + P_{r,i}\ (P_{max} - P_0)$.

<u>Exit of calculation procedure</u>

The calculations stop when the P_r value becomes greater than 0.99. This means that the growing variable came to 99% of saturation. Further calculations are surely senseless because of the model error.

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APPENDIX 2: Source data tables

1. **Complex parameter of dot matrix printers: speed (page/min) * resolution (dpi) * 10-3**

Year	CP (actual)	Model
1977	-	0
1978	0.036	0.012
1979	0.037	0.017
1980	0.073	0.023
1982	0.092	0.045
1983	0.092	0.062
1984	0.092	0.085
1985	0.09	0.11
1986	0.14	0.15
1987	0.20	0.20
1988	0.21	0.26
1989	0.34	0.33
1990	0.46	0.41
1991	0.46	0.50
1992	0.46	0.60
1993	0.71	0.70
1994	0.71	0.81
1995	0.90	0.91
1996	0.83	1.01
1997	1.26	1.10
1998	1.26	1.17
1999	1.30	1.24
2000	1.30	1.30
2001	1.34	1.34

Data source: [15]

2. **Complex parameter of CRT monitors: diagonal (inch) * resolution (pixel/inch^2) * 10^{-3}**

Year	CP	Year	CP
1988	14	1994	67
1989	17	1995	70
1990	22	1996	76
1991	32	1997	78
1992	38	1998	82
1993	50		

Data source: [15]

3. **Maximum clock rate of Intel CPU for PC**

Year	F, MHz	Year	F, MHz
1971	0.108	1996	2000
1972	0.2	1997	300
1974	2	1998	450
1978	10	1999	1200
1979	8	2000	2000
1982	12	2001	3060
1985	32	2002	2530
1988	32	2003	3200
1989	50	2004	3600
1990	25	2005	2200
1991	33	2006	2930
1992	50	2007	3000
1993	200	2008	3200
1994	100	2009	3330
1995	200	2010	3330

Data source: [17]

4. **Efficacy of electric lamps**

Filament			Fluorescent			Arc		
Year	E, Lm/W	Ref.	Year	E, Lm/W	Ref.	Year	E, Lm/W	Ref.
1880	1.5	[2]	1901	10	[2]	1909	15	[25]
1880	2.3	[25]	1906	12	[2]	1913	15	[25]
1881	3.9	[25]	1933	39	[2]	1915	20	[25]
1883	2.5	[16]	1955	72	[2]	1924	20	[25]
1895	2.5	[2]	1960	60	[16]	1926	26	[25]
1896	3.1	[25]	1963	74	[2]	1934	25	[25]
1897	4.0	[2]	1970	70	[16]	1937	31	[25]
1897	4.7	[25]	1975	75	[24]	1938	39	[25]
1900	3.5	[16]	1980	80	[24]	1941	41	[25]
1903	4.7	[25]	1983	83	[24]	1945	41	[25]
1903	7.0	[2]	1987	87	[24]	1949	41	[25]
1903	5.5	[25]	1990	90	[24]	1950	50	[25]
1905	6.0	[2]	1994	92	[24]	1954	50	[25]
1905	10.0	[2]	1998	95	[24]	1960	53	[19]
1907	5.5	[25]	2000	98	[24]	1961	50	[25]
1908	7.8	[25]	2000	100	[16]	1967	100	[25]
1910	6.0	[16]	2005	100	[24]	1972	100	[25]
1911	7.8	[25]	OLED			1973	109	[25]
1911	10.0	[2]	1990	0.5	[24]	1973	116	[25]
1911	10.9	[25]	1995	5	[24]	1980	123	[19]
1913	19.5	[2]	2000	30	[24]	1982	116	[25]
1920	11.0	[16]	2002	70	[24]	1984	126	[25]
1927	32.5	[2]	2005	125	[24]	1996	126	[25]
1931	36.0	[2]				1997	145	[25]
1942	36.0	[2]				2001	145	[25]
LED								
Year	E, Lm/W	Ref.	Year	E, Lm/W	Ref.	Year	E, Lm/W	Ref.
1960	0.1	[16]	1990	10	[24]	2000	100	[16]
1961	0.4	[16]	1992	11	[16]	2001	87	[24]
1975	1	[16]	1994	23	[16]	2002	28	[24]
1982	1.8	[16]	1995	25	[24]	2004	34	[24]
1983	2	[24]	1995	15	[16]	2005	100	[24]
1983	2	[16]	1998	70	[24]	2005	47	[24]
1987	7.5	[24]	1998	30	[16]	2006	65	[24]

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